A function is a rule that sets a correspondence between a domain (D) and range (R). When a set of functions are arranged in such a manner that no two pairs with the same first component in D have the same second component in R, it is considered a one-to-one function. Each element of a set of D is mapped with a unique element of a set of R.

What Is a One-to-One Function?

If f(a) = f(b), then a = b

A function of y = f(x) is one-to-one if each y of the range only exists with exactly one x in the domain related to the y.

The most common method of identifying a one-to-one function is to visualize a horizontal line on the chart. For so long as the function results only intersect with the horizontal line only one time, the function is one-to-one, or .

A bell curve, or parabola, is highly unlikely to ever be one-to-one due to their symmetry resulting in crossing the imaginary horizontal line twice.

Properties of a One-to-One Function

- The domain of f equals the range of f −1 and the range of f equals the domain of f−1.
- f−1(f(x))=x for every x in the domain of f and f(f−1(x))=x for every x in the domain of f −1.
- The graph of a function and the graph of its inverse are symmetric with respect to the line y=x.

Examples

f = {(1 , 2),(3 , 4),(5 , 6),(8 , 6),(10 , -1)}

Two x values, 5 and 8, share a y value of 6. Due to the shared output, this is not a one-to-one function.

a x1 + b = a x2 + b

Once simplified, a(x1-x2) = 0. Since a is to be any value but zero, this leaves x1-x2 = 0, or x1 = x2. So, any linear function where the form f(x) = a x + b , with a ≠ 0, is a one-to-one function.

g (x) = | x - 2 |

This function has some y-values that are paired with more than one of the same x-value, such as (4, 2) and (0, 2). This function is not one-to-one.

f(x) = - x2 + 3

This is easier to disprove. If f(x1) = f(x2), then x1 = x2, which turns the equation into - x1^2 + 3 = - x2^2 + 3. Simplified, -(x1^2 - x2^2) = 0.

Factored, this becomes $-(x_1 - x_2)(x_1 + x_2) = 0$, which leads to $x_1 - x_2 = 0$ or $x_1 + x_2 = 0$. Finally, $x_1 = x_2$ or $-x_2$. Since the graph would be symmetrical from the mid point, it would have two intersections of the imaginary horizontal line test drawn on the graph.

$$f(x) = x^2 - 10x + 15$$

This should be easy. Being the basic format, this formula is clearly a parabola. As discussed earlier, a parabola can not be a one-to-one function.

Inverse Functions

Functions that are one-to-one have an inverse function. Given a function, it may be necessary to find the inverse of that function ($f(x) \rightarrow f^{-1}(x)$). A common method for finding the inverse is to replace $f(x)$ with y. Then, replace every x with a y and every y with an x. Solve for y. Replace y with $f^{-1}(x)$.

Examples

$$\text{Given } f(x) = 3x-2, \text{ find } f^{-1}(x)$$

Following the instructions above, this gives us $y = 3x-2$. Replacing every x with a y and every y with an x gives $x = 3y - 2$. Solving for y takes us to:

$$3y = x+2$$
$$y = x/3 + 2/3$$

Now, replacing y with $f^{-1}(x)$ gives us $f^{-1}(x) = x/3 + 2/3$

Function Disorder

Though calculus can seem a bit overwhelming to many, it doesn't need to be. Any aspect of it can be broken down into smaller sections that just work in tandem with each other, particularly graphing and functions. There are many different styles of functions and formulas in math, with graphing only making each of these more complex. A one-to-one function is just a solitary type of these functions, made unique by its pairing of unique R components for each D component.

Hopefully, this helps explain just what is a one-to-one function to any who were struggling with that idea.